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The Methodology for Calculating Deflections of Reinforced Concrete Beams Exposed to Short Duration Uniform Loading (Based on Nonlinear Deformation Model)

Denis A. Panfilov[a]*, Alexander A. Pischulev[a]

[a] *Samara State University of Architecture and Civil Engineering, Molodogvardeyskaya St, 194, Samara, 443001, Russia*

Abstract

The paper presents an advanced methodology for calculating deflections of reinforced concrete beams with allowance for discrete cracking. The theoretical approach is based on the basic principles of nonlinear deformation model and also takes into account nonlinear concrete and reinforcement behaviour. The described method of calculation is introduced as a computational algorithm worked out on the basis of "MathCAD-15" computer system.
The numerical modelling of reinforced concrete beams being exposed to short duration uniform loading is shown as a finite element model in PC "Lira-9.6R9". The suggested method can be successfully used for calculation of deflections both of reinforced structures made of ultra-high strength concrete (HSC and HPC types) and of normal or over-reinforced structures.

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Keywords: diagrams of concrete deformation; flexural members; deflections; specification documents.

1. Introduction

The methodology for calculating deflections of reinforced concrete flexural members is inextricably connected with the definition of real stress and strain state in any section of the member, considering along the whole structure, cracking detection as well as physical properties of reinforced concrete. Thus, the accuracy and convergence of research results depend on the introduction to the calculation method of realistic deformation models of concrete and reinforcement.

The purpose of this work, as continuation of research conducted [1-5], is to improve methods for calculating statically determinate flexural reinforced concrete beams deflections under uniformly distributed loading, taking into account the discrete cracking and the nonlinear behaviour of reinforcement and concrete.

Corresponding author. Tel.:+7-909-343-1986
E-mail address: panda-w800i@ya.ru

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Nomenclature

σ_b (ε_{bx})	compressed concrete stress
σ_{bt} (ε_{bty})	tensile concrete stress
$f_s(\varepsilon_s)$	tensile reinforcement stress
f_{sc} (ε_{sc})	compressed reinforcement stress
ε_{bx}	current value of strain in compressed concrete
ε_{by}	current value of strain in tensile concrete
ε_{sc}	current value of strain in compressed reinforcement
ε_s	current value of strain in tensile reinforcement
ε_{b2}	maximum value of strain of compressed concrete
ε_{bt2}	maximum value of strain of tensile concrete
x	present altitude of compressive zone of the concrete
y	present altitude of tensile zone of the concrete
k	altitude of compressive zone of the concrete
t	altitude of tensile zone of the concrete
h	overall depth of section
b	width of section
a_1 and a_2	depth of concrete cover of tensile and compressive reinforcement
A_s and A_{sc}	tensile and compressive reinforcement area

2. Description of an advanced method for calculating deflections of flexural reinforced concrete members

There are many different methods to determine the deformation dependence of compressed and tensile concrete and reinforcement described in domestic and foreign scientific works and specification documents.

As a basis of concrete tensile strain diagram we accepted the diagram of G.V. Murashkin [1] using research of G.A. Smolyago [9].

Such methods as Eurocode 2 [8] for reinforcement without yield line, Wang et al [10] for reinforcement with yield line, are accepted as deformation diagram of compressed and tensile reinforcement.

Based on the deformation diagram proposed in SSUACE an advanced method for calculating deflection of flexural reinforced concrete structures was developed.

In this case the following assumptions are accepted:

- There is flexural reinforced concrete simply supported beam loaded by two concentrated force
- Equispaced from bearings under consideration;
- There are two characteristic sections along the length of a beam- a section (I) without cracks in the zone of the lateral force action and a section (II) with cracks in pure flexure zone;
- Curvature distribution in the section (I) is made linearly;
- Curvature distribution in the section (II) is made in a sinusoidal manner of variable amplitude.
- Sinusoidal law characteristics are determined by calculating the height of the compressed zone in the area of cracks and blocks between cracks;
- Curvature R - in a section with cracks is determined by the equilibrium condition in the compressed concrete efforts and tensile reinforcement;
- Curvature R1 - in a section in the middle of the block between cracks- is determined from the condition that strain values of tensile concrete do not exceed the limit (if $\varepsilon_{bt}= \varepsilon_{bt2}$), and the hypothesis of planar sections is applied only to compressed concrete.
- Algorithm for determining the deflection of reinforced concrete flexural structures exposed to short term duration uniform loading has been implemented in *MathCAD 15*.

Equilibrium equations in a block section without cracks will be as follows (1):

$$\begin{cases} \int_0^k \sigma_b\left(\frac{\varepsilon_b}{k}\cdot x\right)\cdot b\cdot dx - \int_0^t \sigma_{bt}\left(\frac{\varepsilon_{bt}}{t}\cdot y\right)\cdot b\cdot dy - f_s\left[\frac{\varepsilon_{bt}}{t}\cdot\left(h_0-k\right)\right]\cdot A_s + \\ + f_{sc}\left[\frac{\varepsilon_b}{k}\cdot\left(k-a_2\right)\right]\cdot A_{sc} = 0 \ ; \\ \int_0^k \sigma_b\left(\frac{\varepsilon_b}{k}\cdot x\right)\cdot b\cdot dx\cdot\left[\left(\frac{\int_0^k \sigma_b\left(\frac{\varepsilon_b}{k}\cdot x\right)\cdot x\cdot dx}{\int_0^k \sigma_b\left(\frac{\varepsilon_b}{k}\cdot x\right)\cdot dx}\right)+h_0-k\right]+ \\ + f_s\left[\frac{\varepsilon_b}{k}\cdot\left(k-a_2\right)\right]\cdot A_{sc}\cdot\left(h_0-a_1-a_2\right)- \\ -\int_0^t \sigma_{bt}\left(\frac{\varepsilon_{bt}}{t}\cdot y\right)\cdot b\cdot dy\cdot\left(h\ 0-k-\frac{\int_0^t \sigma_{bt}\left(\frac{\varepsilon_{bt}}{t}\cdot y\right)\cdot y\cdot dy}{\int_0^t \sigma_{bt}\left(\frac{\varepsilon_{bt}}{t}\cdot y\right)\cdot dy}\right)-M_u=0, \end{cases} \tag{1}$$

By solving equations (1) we obtain the values of k and ε_b considering nonlinear deformation of concrete and reinforcement for tension and compression with the appropriate strain diagrams. Hence the curvature R is defined by:

$$R=\frac{\varepsilon_b}{k} \tag{2}$$

Stress and strain state in the middle of the block separated by cracks are described by:

$$\begin{aligned} &\int_0^{k1} \sigma_b\left(\frac{\varepsilon_{b1}}{k_1}\cdot x\right)\cdot b\cdot dx - \int_0^{H-k1} \sigma_{bt}\left(\frac{\varepsilon_{bt2}}{h-k_1}\cdot y\right)\cdot b\cdot dy - f_s\left[\frac{\varepsilon_{b1}}{k_1}\cdot\left(h_0-k_1\right)\right]\cdot A_s + \\ &+ f_{sc}\left[\frac{\varepsilon_{b1}}{k_1}\cdot\left(k_1-a_2\right)\right]\cdot A_{sc} = 0 \end{aligned} \tag{3}$$

As the distance value between cracks l_s in the algorithm it is possible to choose different methods, such as SP [6] or EuroCode [8]. In this work, we used the l_s values, obtained by EuroCode method [8].

The functions of the curvature variation at one of the block sites between cracks are taken using the sinusoidal equation (4) with different amplitude depending on the area under consideration:

$$P_{33`}(x)=\frac{R_{m3}-R`_{m3}}{2}\cdot\sin\left(\frac{-2\pi}{l_s}\cdot x+\frac{\pi}{2}+\frac{2\cdot\pi\cdot m_3}{l_s}\right)+\frac{R_{m3}+R`_{m3}}{2} \tag{4}$$

Curvature variation graph at $m_3 – m_{3`}$ is shown in Figure1 below:

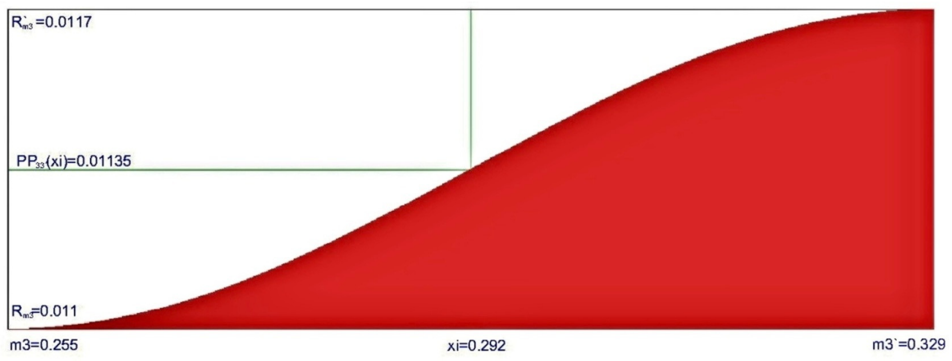

Fig. 1. Curvature variation graph at $m_3 – m_{3`}$.

General diagram of the curvature variation along the member length is shown in Figure 2:

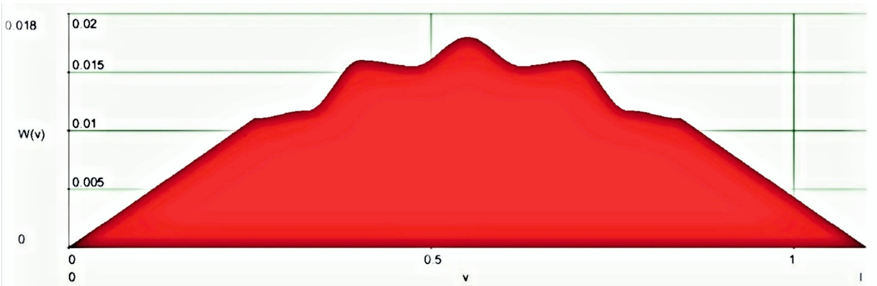

Fig. 2. The diagram of the curvature variation along the member length.

The total deflection calculation of a beam is conducted under the general rules of structural mechanics taking into account the resulting change in the sectional curvature diagram $W(v)$ along the beam indicated in Figure 2.

3. Numerical modelling, comparison of the results to the existing methods of calculation

As reliability evaluation of suggested methods numerical experiment has been conducted in nonlinear formulation for the finite element model in PC "Lira 9.6R9".

The computational model consists of flat quadrilateral finite elements, the upper and lower longitudinal and lateral reinforcement in the form of rod finite elements. Cracks were modelled by jointing nodes. The distance between cracks was taken based on the calculation procedure of Eurocode 2 [8].

The geometrical and physical characteristics of the test sample are shown in Tables 1 and 2.

As a result of conducted research the correctness of sinusoidal function usage is visually confirmed for the distribution of the neutral axis and the curvature (Figure 3):

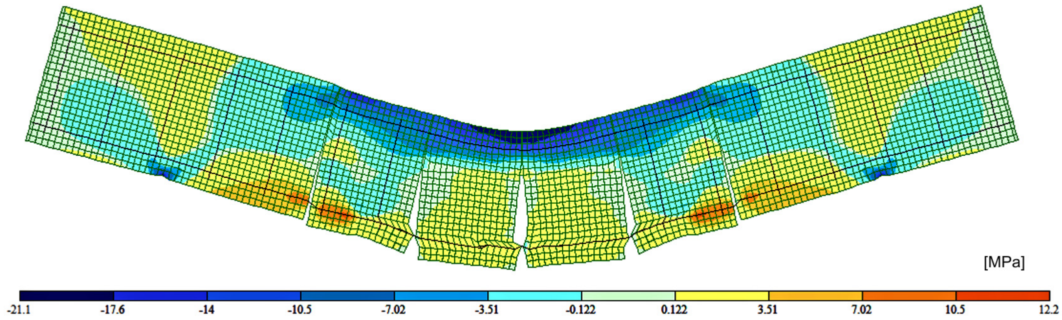

Fig.3. Diagram of the stress distribution in concrete.

Table 1. Basic characteristics of the specimen under consideration.

Geometrical characteristics				Reinforcement			Reinforcement ratio	
Specimen designation	Bearing distance, m	Depth of section, m	Width of section, m	Tensile reinforcement	Compressed reinforcement	Lateral reinforcement	Tensile reinforcement,%	Compressed reinforcement, %
Ob 1-3.3	1.1	0.22	0.12	1Ø20	1Ø10	Ø10s90	1.189	0.296

Table 2. Materials physical properties of the specimen under consideration.

	Properties of concrete				Properties of reinforcement				
Specimen designation	Concrete compres-sive strength, MPa	Concrete tensile strength, MPa	Ultimate compres-sive strain of concrete	Ultimate tensile strain of concrete	Area of longitudi-nal working reinforce ment, mm^2	Elastic limit, MPa	Tensile strength, MPa	Yield strain of reinforce-ment	Reinforce-ment strain at rupture
Ob 1-3.3	21.74	1.8	0.0034	1.26e-4	314.0	477.0	646.5	2.371e-3	0.069

In order to estimate the reliability of obtained results, there was a comparison conducted with methods of calculation based on specification documents SP 63.13330.2012 [6], guides to SP 52-101-2003 [7], EuroCode 2 [8] and also to results of nonlinear analysis in the finite element model with discrete cracks pattern in PC "LIRA9.6R9."

Calculation results are given in Table 3. and convergence estimates of results are given in Table 4.

Table 3. Research results summary

Series marking	Values of deflection by this method, mm				
	Developed method	SP 63.13330.2012	Guide to SP 52-101-2003	Eurocode 2	Nonlinear analysis of PC «LIRA9.6R9»
Ob 1-3.3	2.004	2.932	2.131	2.27	1.91

Table 4. Research results summary

Series marking	Variation in deflection by this method from the numerical experiment value in PC «LIRA9.6R9», %			
	Developed method	SP 63.13330.2012	Guide to SP 52-101-2003	Eurocode 2
Ob 1-3.3	4.92	53.5	11.5	18.85

4. Conclusions

Based on the results of the research conducted, it is possible to draw conclusions on the highest convergence of results of structural deflection using the suggested method. The developed method of calculation with allowance for discrete cracking, based on nonlinear deformation model of concrete, allows to estimate more precisely deflections of flexural reinforced concrete structures in comparison to current SP [6] and its guide [7]. Future research will be directed to conduct experimental full-scale test in order to confirm the suggested method of calculation.

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